Computer Science G12

Boolean Logic

Assignment 4. Due date: Fri. 27 Oct. 2017

See examples below. Note p’,!p or ~p, all mean NOT p.

1. Write the truth table for the biconditional: p <-> q
2. Prove p = p.q + p.(~q)
3. From p->q and q->p, conclude p’.q’+qp using Truth Table
4. Idem, but prove it algebraically
5. Find equivalent expression w/o any conditional nor biconditional : (q<->p) + (p=>q)
6. Prove using TT: (p->q) (~q) -> p
7. Idem, algebraically
8. Prove using TT: X.(X+Y) = X
9. Idem, algebraically
10. Write the dual expression of the following ones:
11. p.(!p+q)
12. (X+Y).(X+!Y).(!X+Y)
13. (A.1)+(A+0+~A)
14. (A+B).(~A+B)
15. ABC+ A(!B)C + (!A)B(!C)
16. From p->q and q->p, conclude algebraically q’+qp
17. From p->q and q->p, conclude algebraically p’+qp
18. Find the complement of x.(y’.z’+y.z)
19. Simplify using the laws of Boolean algebra: x.y+x’.z+y’.z

Example: Let’s prove algebraically that, given p->q and q->p, we can conclude ~p+qp. Remember that “=>” means “from the expression on the left we can conclude the expression on the right”, and viceversa for the arrow in the other sense.

From the statement of the problem we assume we can claim

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1. (p->q).(q->p)  (problem assumption)
2. (p->q)(from 1 and definition of conjunction)
3. ~p+q  (from 2. and definition of the conditional operator)
4. ~{p.(~q)} (from 3 and De Morgan laws)
5. ~{[p.(~q)] + F} (from 4 and definition of conjunction and definition of F)
6. ~{[p.(~q)] + (~p.p)} (from 5 and definition of conjunction )
7. ~{p.[(~q)+(~p)]} (from 5 and distributive laws; also from 5 in CNF )
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8. ~p+qp ( from 6 and de Morgan Laws)
q.e.d
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The sequence of steps above means that we claim to be true that "step i => step i+1", and we argue so for the reason in parenthesis given on the right of each step.

Another way to prove it is starting from the 7 and reaching up to 2. From there to 1 requires only the additional assumption of (q->p) which is given in the problem statement. This would finish this other proof.